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TECHNICAL REPORT 231-10

A COMPARISON BETWEEN THEORETICAL AND MEASURED WAVES ABOVE A SUBMERGED RANKINE BODY

By

C. C. Hsu

And

B. Yim

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NOTATION

f	Depth of submergence measured from the centerline of the body
F	Froude number based on depth
g	Gravitational acceleration
M	Strength of point source
R	Radial distance from source or sink
t	Time
U	Uniform free stream speed
x,y,z	Rectangular cartesian coordinates
ζ	Surface elevation
ζ_r	Regular wave height
ζ_ℓ	Local disturbance
μ	Fictitious frictional constant

SUMMARY

A comparison is made of theoretical and measured wave patterns produced by a submerged Rankine body (7:1) moving at constant speed. The measurements have been made by the staff of the David Taylor Model Basin, for a range of Froude numbers, but for a single depth/length ratio (1:3). Calculations were made for both the far and near fields. The former utilize the stationary phase approximation and the latter involve numerical integrations and include the local disturbance. In general, the agreement between theory and experiment is as good as the agreement between measurements taken at the same Froude number but for two different body lengths (4.5 and 9.0 feet). It is concluded from these tests that first-order wave theory predicts the shape and amplitude of the waves produced by a fully submerged body with sufficient accuracy for almost all practical purposes.

INTRODUCTION

In the past few years there has been a renewed interest in the problem of determining the surface wave pattern due to submerged bodies. Considerable progress in both the theoretical and experimental area has since been made. For a simple submerged body travelling at constant speed in deep water, an extensive analysis has been given by Yim (1). More recently, surface wave height measurements due to a Rankine ovoid in a towing tank have been undertaken at the David Taylor Model Basin (2), (3).

The object of this report is to compare the results between measured values and theoretical calculations.

The surface disturbance produced by a submerged body is usually analyzed into two parts: One is called the local disturbance which is, more or less, symmetrical and moves with the body; the other forms a group of regular waves which travels with group velocity within a vee-shaped region behind the body. In the case of towing tank experiments, there is an additional time-dependent disturbance which travels both upstream and downstream of the body due to its starting conditions. The fluid motion due to submerged bodies started from rest has been discussed in some detail in (4); the resulting surface disturbance is shown in Figure 1. The steady regular waves, in this case, travel downstream from the origin to $x = (1/2)Ut$ with group velocity $U/2$. It has been found that the part due to the initial acceleration diminishes rather rapidly with time. The effect of the unsteady disturbance, for sufficiently long time t, becomes appreciable, in general, only far downstream, i.e., $x \rightarrow (1/2)Ut$. Since the steady wave pattern due to a submerged Rankine ovoid is of primary concern here, the theoretical and experimental comparisons are limited to those tests which have unimportant unsteady effects.

The wave pattern of a submerged Rankine ovoid in a uniform stream, to a first approximation, may be calculated as due to distribution of a point source and point sink. The surface disturbances in the near field, are generally evaluated by numerical

integrations. The regular waves, in the far field, say 3 or 4 body-lengths away from the body, can easily be estimated by applying the method of stationary phase. In Figures 3 and 4, the test data of a 4.5 feet long, 7 to 1, Rankine ovoid towed at 10, 9, 7.3 and 6 feet per second and a given submergence depth of 1.5 feet (Reference 2) together with the corresponding calculations are shown. The calculated curves have the same general shape as the experimental measurements; the agreement between theory and experiment is discussed later. The centerline wave profile measurements of a 9 feet long, 7 to 1, Rankine ovoid towed at 10 feet per second and a submergence depth of 3 feet (Reference 3) is also examined. Except that the physical scale of the body length and submergence depth were doubled, this test was otherwise conducted under very similar conditions as that given in (2) for the case U = 7.3 feet per second. Comparison of non-dimensional theoretical calculations and experimental centerline wave profiles due to the two different length-scale models are shown in Figure 5. It is found that the theoretical calculations are in much better agreement with the larger model test data.

THEORETICAL CONSIDERATIONS

When a Rankine ovoid is submerged in a uniform stream, the fluid motion, to a first approximation, may be taken to be that due to a point source and point sink of constant strength $|M|$ distributed at points $(0, 0, -f)$ and $(\ell, 0, -f)$ respectively as shown in Figure 2. If the fluid is assumed to be inviscid,

incompressible and infinite in extent, the surface elevation in this case may be shown approximately to be

$$\zeta = -\sum_{n=1}^{2} \frac{M_n}{2\pi U} R\ell \int_{-\pi}^{\pi}\int_{0}^{\infty} \frac{i\, 2\kappa \sec\theta\, e^{\kappa(i\omega_n - f)}}{\kappa - \kappa_o \sec^2\theta - i\mu \sec\theta}\, d\kappa\, d\theta \qquad [1]$$

in which

$$\kappa_o = \frac{g}{U^2}$$

$$M_1 = M \quad , \qquad M_2 = -M_1 = -M$$

$$\omega_1 = x \cos\theta + y \sin\theta = (R_1) \cos(\theta - \delta_1)$$

$$\omega_2 = (x - \ell) \cos\theta + y \sin\theta = (R_2) \cos(\theta - \delta_2)$$

$$R_1 = \sqrt{x^2 + y^2} \quad , \qquad R_2 = \sqrt{(x-\ell)^2 + y^2}$$

$$\delta_1 = \tan^{-1} \left(\frac{y}{x}\right) \quad , \qquad \delta_2 = \tan^{-1} \left(\frac{y}{x-\ell}\right)$$

and μ is a fictitious frictional constant. The integral in Equation [1] is transformed by contour integration; when μ is made zero ultimately, the complete expressions are

$$\zeta = -\frac{2}{\pi}\sum_{n=1}^{2}\frac{M_n}{Uf}\int_{-\pi/2+\delta_n}^{\pi/2+\delta_n}\int_0^{\infty}\frac{\exp(-m\omega_n)\, m \sec\theta}{\kappa_o^2\sec^4\theta + m^2}\left[\kappa_o f \sec^2\theta \sin(mf) - mf\cos mf\right] dm\, d\theta$$

$$+ 4\sum_{n=1}^{2}\frac{M_n}{U}\kappa_o\int_{-\pi/2+\delta_n}^{\pi/2}\exp(-\kappa_o f \sec^2\theta)\sec^3\theta\cos(\kappa_o\omega_n\sec^2\theta)d\theta \quad [2]$$

The first sum in Equation [2] is the local disturbance ζ_ℓ, which diminishes with large $|x|$. The second sum represents the regular wave which travels only downstream and is contained within two straight lines radiating from the origin (0, 0, 0), each making with the line of motion an angle, δ_c, approximately $19^\circ 28'$.

Near the body the evaluation of ζ is not simple and can only be done numerically; the details of the calculations have been discussed in (1). Far behind the body Equation [2] may be expressed as

$$\zeta \approx \zeta_r = 4\sum_{n=1}^{2}\frac{M_n}{U}\kappa_o\int_{-\pi/2}^{\pi/2}\exp(-\kappa_o f\sec^2\theta)\sec^3\theta\cos(\kappa_o R_n F_n(\theta))d\theta, R>>\ell$$

[3]

with $F_n(\theta) = \sec^2\theta\cos(\theta - \delta_n)$. By applying the method of stationary phase, Equation [3] becomes

$$\zeta = 4\varkappa_o \sum_{n=1}^{2} \frac{M_n}{U} \sec^3\theta_{n1} \exp(-\varkappa_o f \sec^2\theta_{n1}) \sqrt{\frac{2\pi}{\varkappa_o R_n |F_n''(\theta_{n1})|}} \cos\left(\varkappa_o R_n F_n(\theta_{n1}) + \frac{\pi}{4}\right)$$

$$+ 4\varkappa_o \sum_{n=1}^{2} \frac{M_n}{U} \sec^3\theta_{n2} \exp(-\varkappa_o f \sec^2\theta_{n2}) \sqrt{\frac{2\pi}{\varkappa_o R_n |F_n''(\theta_{n2})|}} \cos\left(\varkappa_o R_n F_n(\theta_{n2}) - \frac{\pi}{4}\right) \qquad [4]$$

where

$$\theta_{n1} = -\frac{1}{4}\left[\cot\delta_n - \sqrt{\cot^2\delta_n - 8}\right]$$

$$\theta_{n2} = -\frac{1}{4}\left[\cot\delta_n + \sqrt{\cot^2\delta_n - 8}\right]$$

The first and second sum in [4] represent systems of transverse and diverging waves respectively travelling with the body; the amplitude for a given azimuth δ diminishes as $(k_oR)^{-\frac{1}{2}}$. On the centerline, where $\delta_n = 0$, Equation [4] reduces to

$$\zeta \doteq \zeta_r = \zeta_{r)}\ \text{transverse}$$

$$= 4\varkappa_o \sum_{n=1}^{2} \frac{M_n}{U} \exp(-\varkappa_o f) \sqrt{\frac{2\pi}{\varkappa_o R_n}} \cos(\varkappa_o R_n + \frac{\pi}{4}) \qquad [5]$$

In the next section comparisons between the calculated values and experimental data are made.

COMPARISONS AND DISCUSSIONS

The main sets of experimental results are taken from a DTMB test report by Shaffer (2). In these tests a 4.5 feet long, 7 to 1, Rankine ovoid was towed at constant speed U and a given submergence depth, f = 1.5 feet, in the 360 feet long, 240 feet wide and 20 feet deep basin. The length of the test run was limited to 225 feet. The wave profiles were measured at five stations; approximately 48, 63, 95, 133 and 187 feet from the starting position of the model. The influence of the initial acceleration, as discussed in (4), diminishes rather rapidly as the running time t increases and becomes appreciable, for sufficiently long time, only far downstream. The starting conditions, except far downstream, i.e., $x \rightarrow (1/2)Ut$, would, in general, have little effect on the measurements made at the last two stations. The last (187 feet) measuring station, however, was near the end of the run, so that the major part of the measurements for this station may be affected by the deceleration of the model. Therefore, only the data measured at the 133 feet station are being used in the comparison between the experimental and the theoretical steady wave profile calculations.

The measured centerline wave profiles in test runs at U = 10.0, 9.0, 7.3 and 6 feet per second together with the corresponding calculated values are shown in Figure 3(a), (b), (c) and (d) respectively. It can be seen that the calculated curves

express the general variations in the same manner as the experimental measurements. The agreements are generally better at higher speeds or higher Froude numbers $F(= U/\sqrt{g\ell})$. It is to be noted that in these comparisons, the experimental reference point P near the body, at which $\zeta = 0$, is taken to be coincident with that of the calculated zero wave height point between $x = 0$ and ℓ. The exact location of point P depends very much on the running speed U. It is found from the calculations that P is close to $(1/2\ \ell, 0, 0)$, as postulated in (2) for all test speeds, only at F near 1. The exact behavior of this phase difference as a function of Froude number may be worth further experimental investigation.

The off centerline wave profile measurements in these tests were made at y = 11.375, 22.750, and 34.125 feet. The regular wave pattern begins to develop only after $x \approx y/\tan(19^\circ 28')$ which is quite far downstream in the latter cases; the measurements at y = 22.750 and 34.125 feet are largely distorted by the transient effects before the steady portion of the wave train can be developed. The only meaningful experimental data which can be compared with the theoretical calculation is that measured at y = 11.375 feet. In Figure 4 comparison of theoretical and experimental off centerline wave profiles at U = 7.3 feet per second, y = 11.375 feet is given; the results are in fairly good agreement.

Before discussing these data further, a different set of experimental results, taken from a previous DTMB test report (3), should be examined. In this test, a 9 feet long, 7 to 1, Rankine

ovoid was towed at U = 10.0 feet per second and f = 3 feet. Except that the physical scale of the body length and submergence depth were doubled, the experiment was otherwise conducted under very similar test conditions as that given in (2) for the case U = 7.3 feet per second and f = 1.5 feet. The non-dimensional centerline wave height, $\zeta/(M/Uf)$, versus x/ℓ for both the experiments and their corresponding calculations are shown in Figure 5. The theoretical calculations, as seen from Figure 5, are in much better agreement with the data given in (3) than that in (2). The measured nondimensional wave amplitude for the 9 foot body is generally 15 percent higher than that for the 4.5 foot body. This discrepancy may not be due entirely to instrumentation difficulties, since the two tests were conducted at similar Froude but quite different Reynolds scales.

In general the theoretical predictions are in fairly good agreement with the test results. At the higher Froude numbers and for the larger test body, the agreement between the predicted and measured wave lengths and amplitudes behind the body are excellent. In all cases the shape of the wave over the body is well predicted by the theory, but the amplitude of both the first crest and the first trough are overestimated, sometimes by a factor of 2. The reason for this is not clear, but it is to be noted that in the case of the large body (9 feet length) at 10 feet per second the agreement between the theory and experiment for the first trough amplitude is really excellent, while the first peak is overestimated by only about 25 percent.

We should imagine that as far as the wave theory itself is concerned, that the agreement between theory and measurement might become worse with increasing Froude number, as the effect of the surface waves on the submerged body become more pronounced; this is, however, not observed to be the case. In fact, the test results may be interpreted to suggest that for too small scales and speeds, viscous effects in the flow about the body begin to influence the wave pattern.

In any case, it seems safe to conclude that existing wave theory predicts the shape and magnitude of the waves above a fully submerged body with sufficient accuracy for almost all practical purposes.

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4. Hsu, C. C., "On the Surface Wave Pattern of Submerged Bodies Started from Rest," HYDRONAUTICS, Incorporated Technical Report 231-7, April 1965.

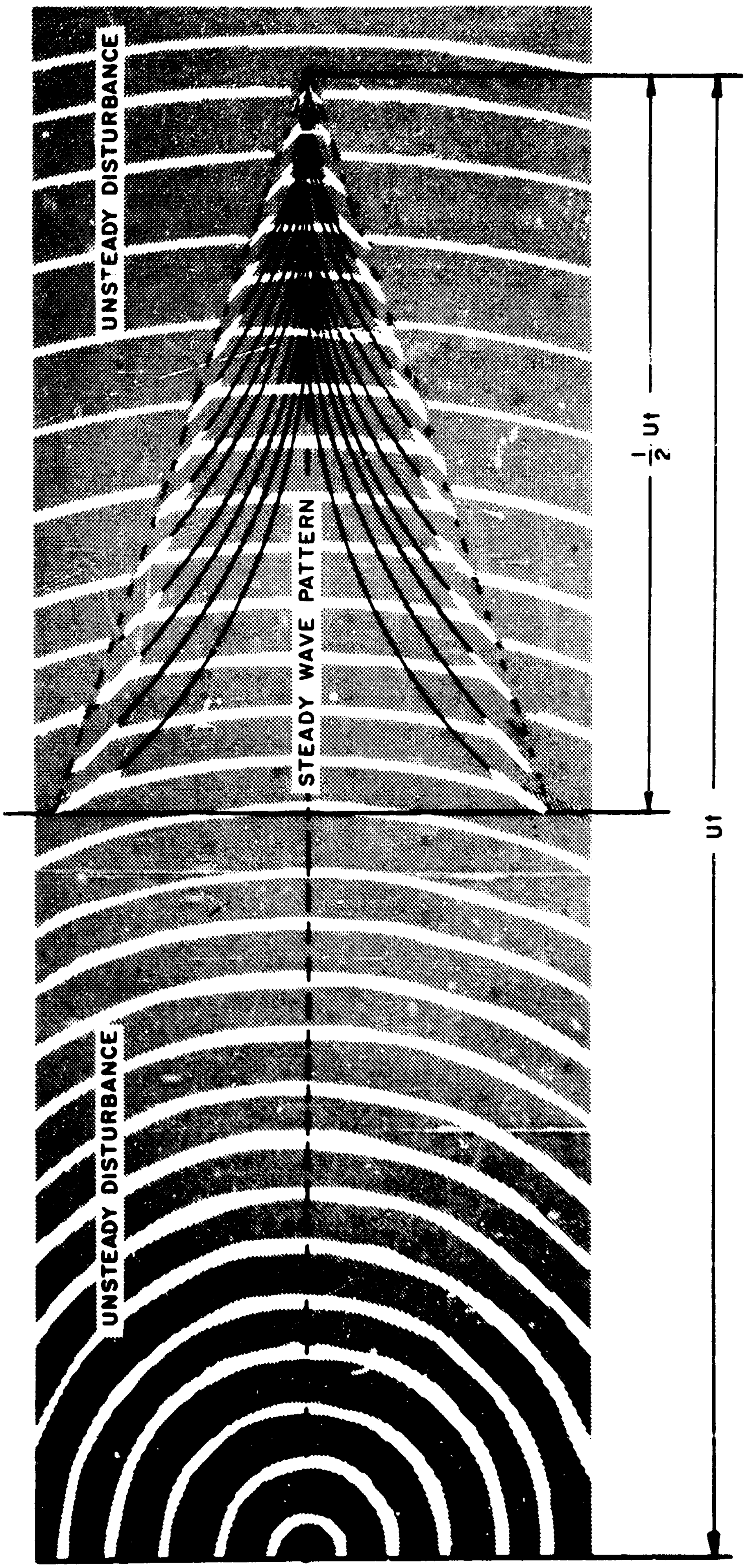

FIGURE 1 -SURFACE WAVE PATTERN OF A SUBMERGED BODY STARTED IMPULSIVELY FROM REST

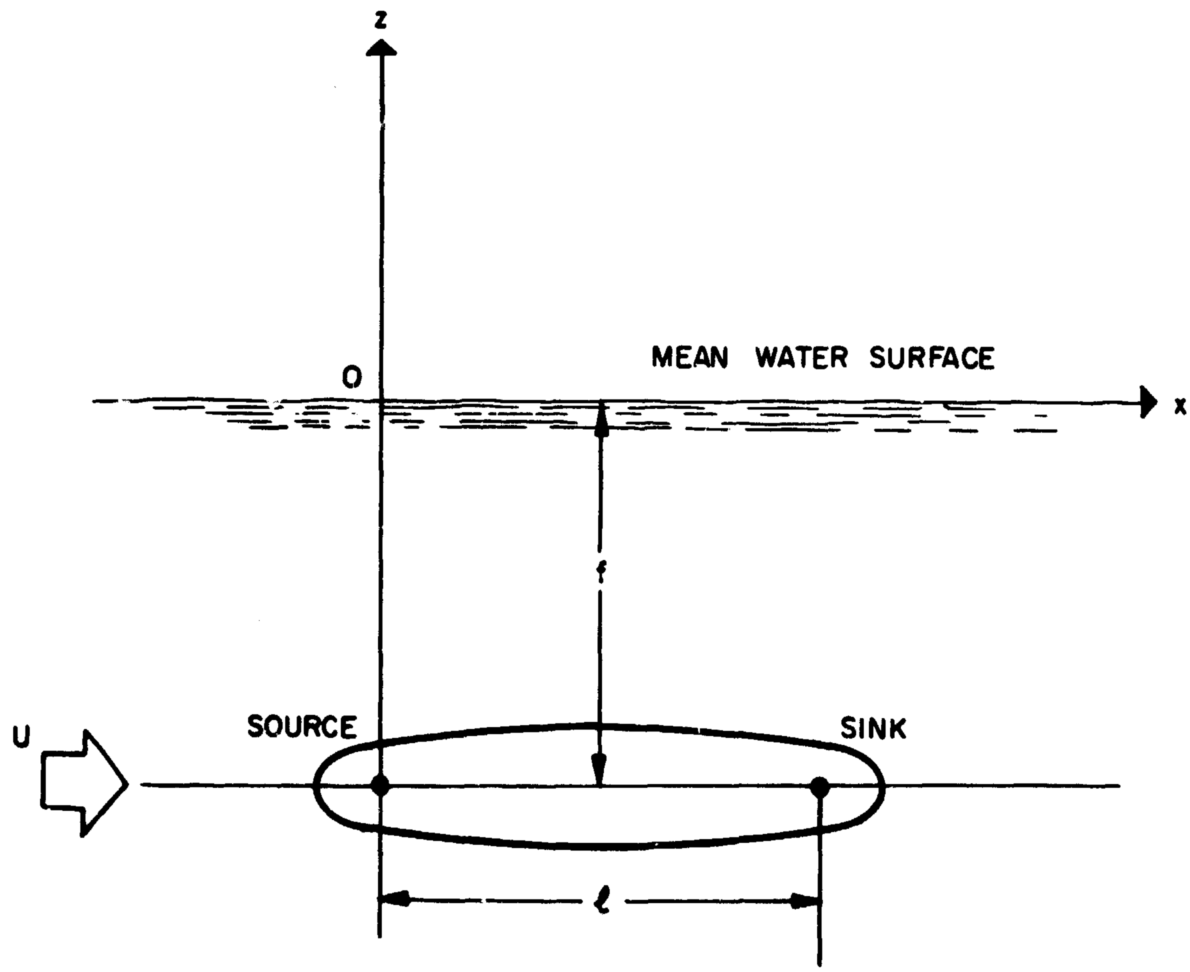

FIGURE 2– DEFINITION SKETCH

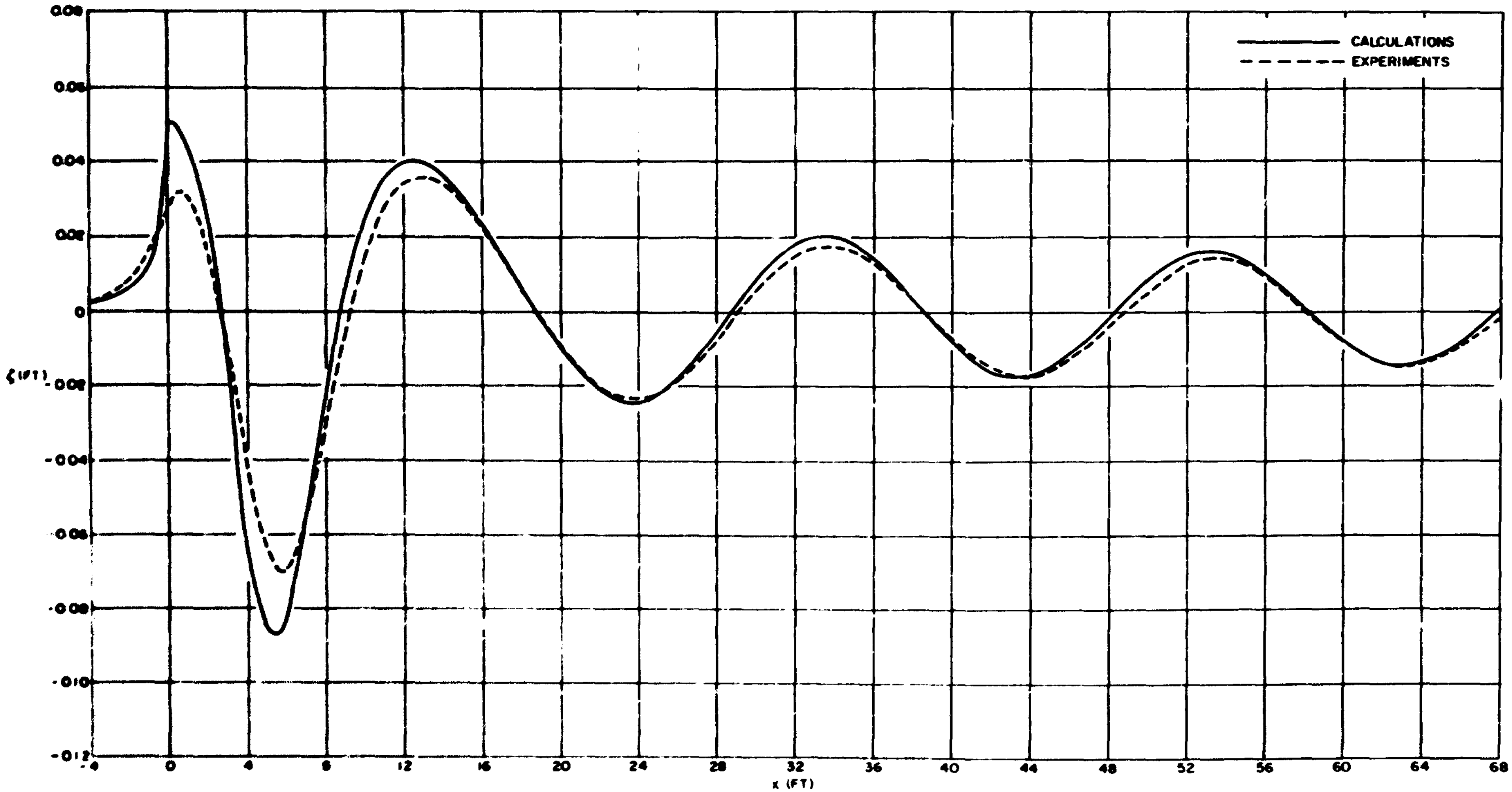

FIGURE 3(a) - COMPARISON OF THEORETICAL AND EXPERIMENTAL CENTERLINE WAVE PROFILES DUE TO A 4.5 FT, 7 TO 1, RANKINE OVOID SUBMERGED 1.5 FT BELOW THE FREE WATER SURFACE U=10 FT/SEC

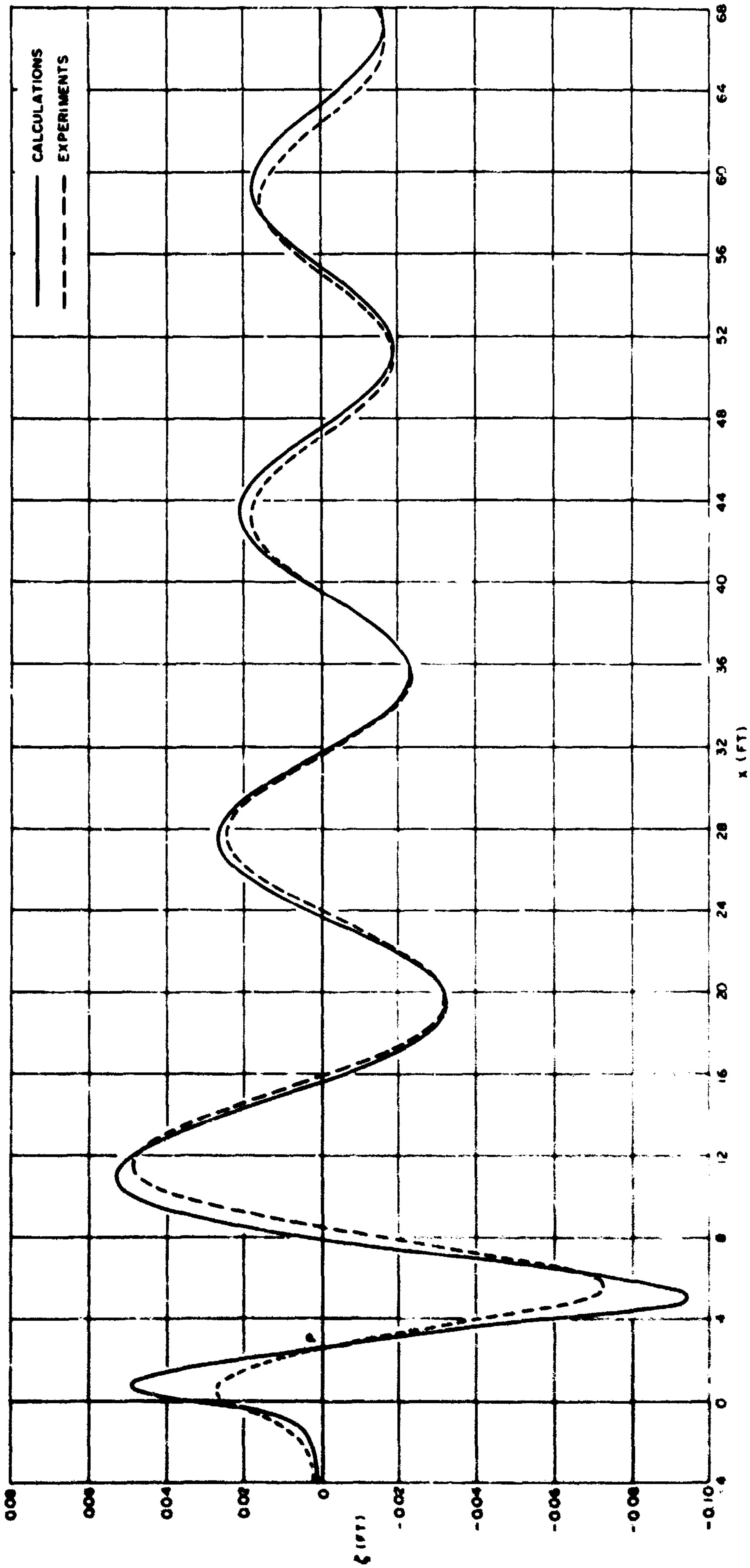

FIGURE 3(b) – CONTINUED U = 9.0 FT/SEC

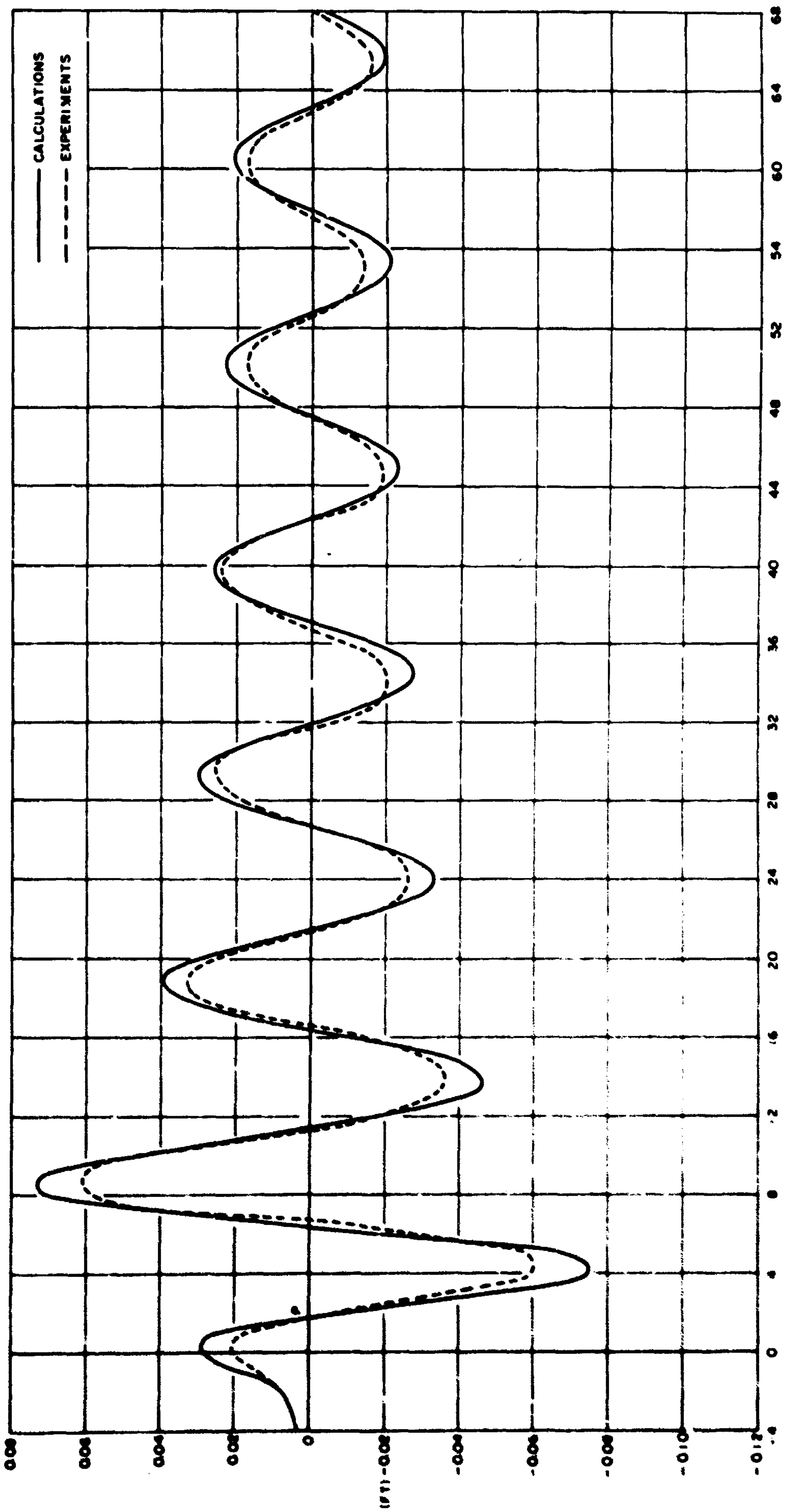

FIGURE 3(c) - CONTINUED U = 7.3 FT/SEC

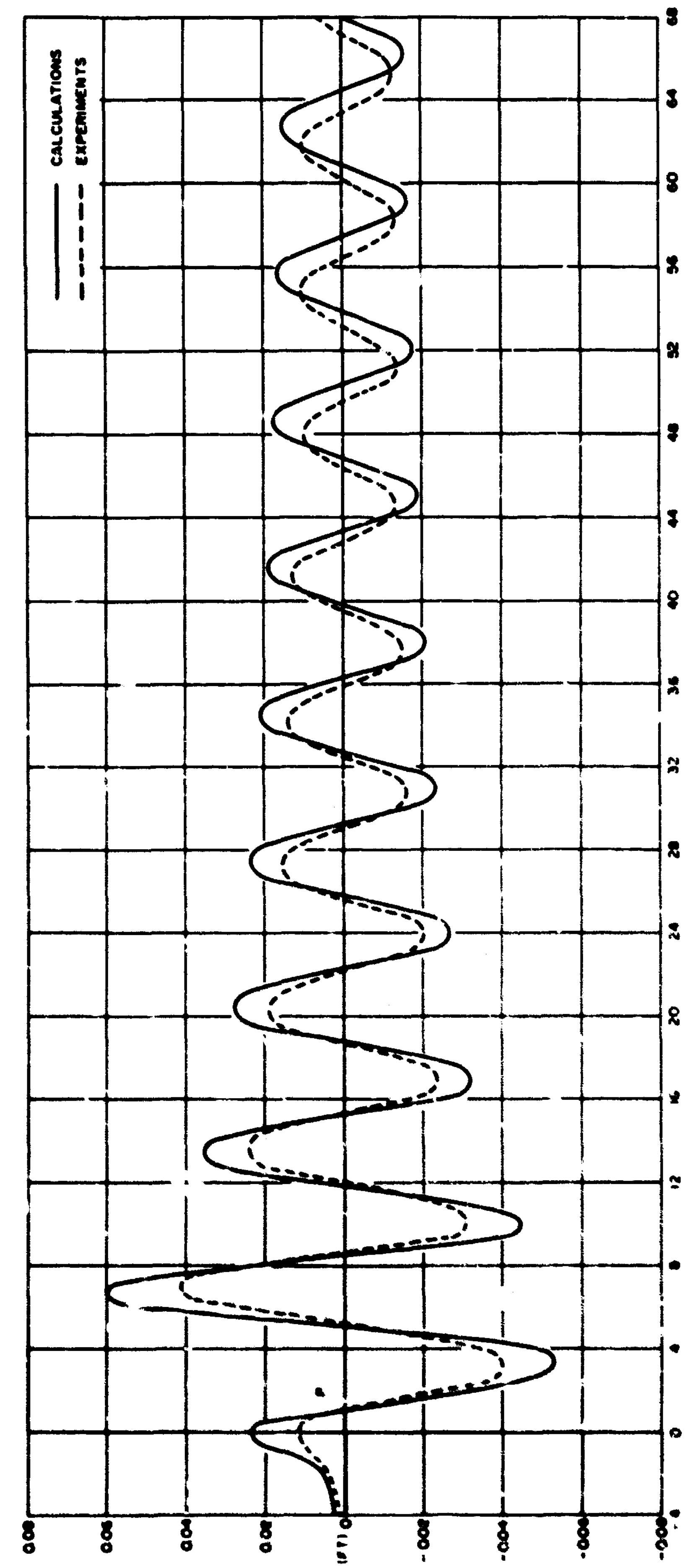

FIGURE 3(d) - CONCLUDED U = 6.0 FT/SEC

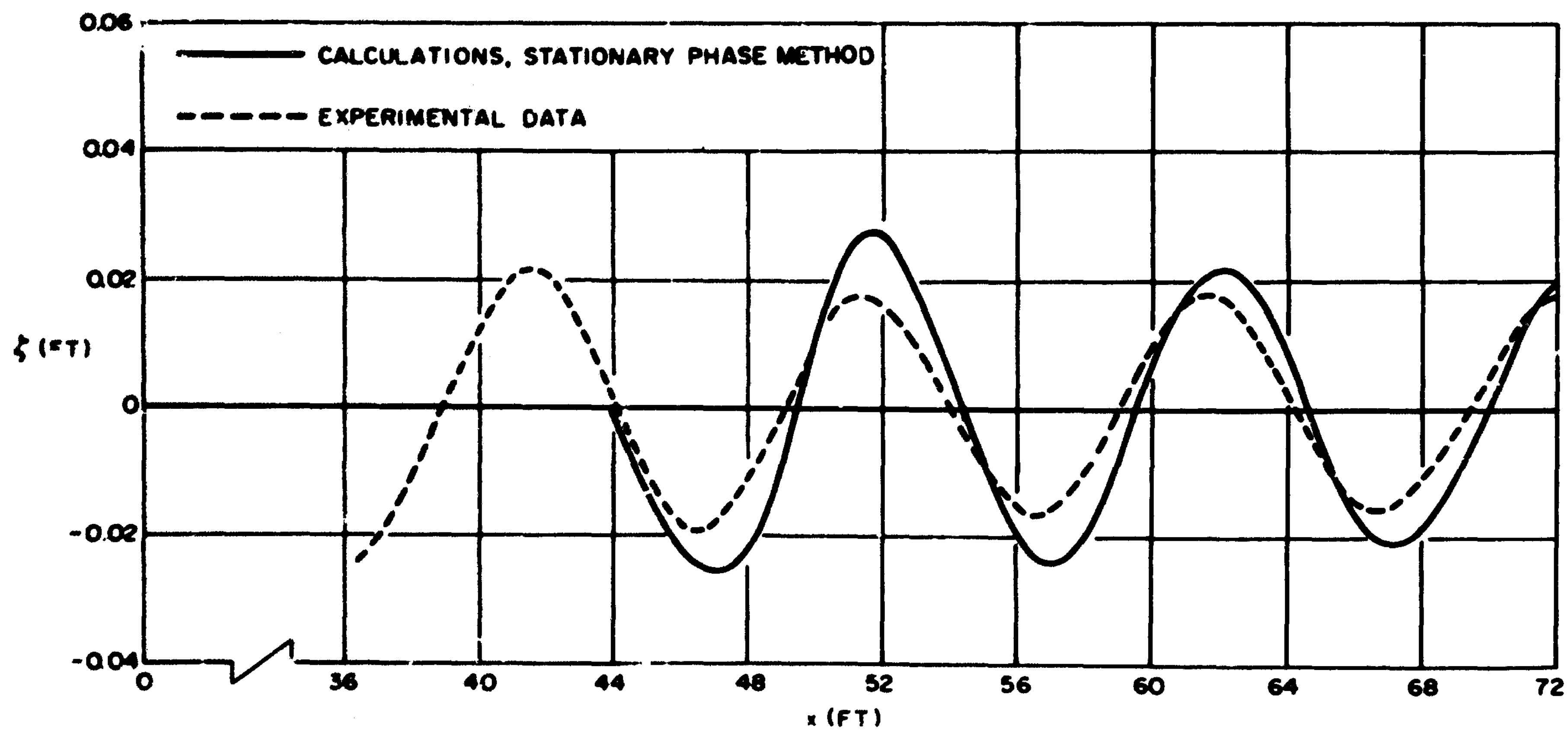

FIGURE 4– COMPARISON OF THEORETICAL AND EXPERIMENTAL OFF CENTERLINE WAVE PROFILE DUE TO A 4.5 FT, 7 TO 1, RANKINE OVOID SUBMERGED 1.5 FT BELOW THE FREE WATER SURFACE

U = 7.3 FT./SEC. y = 11.375 FT.

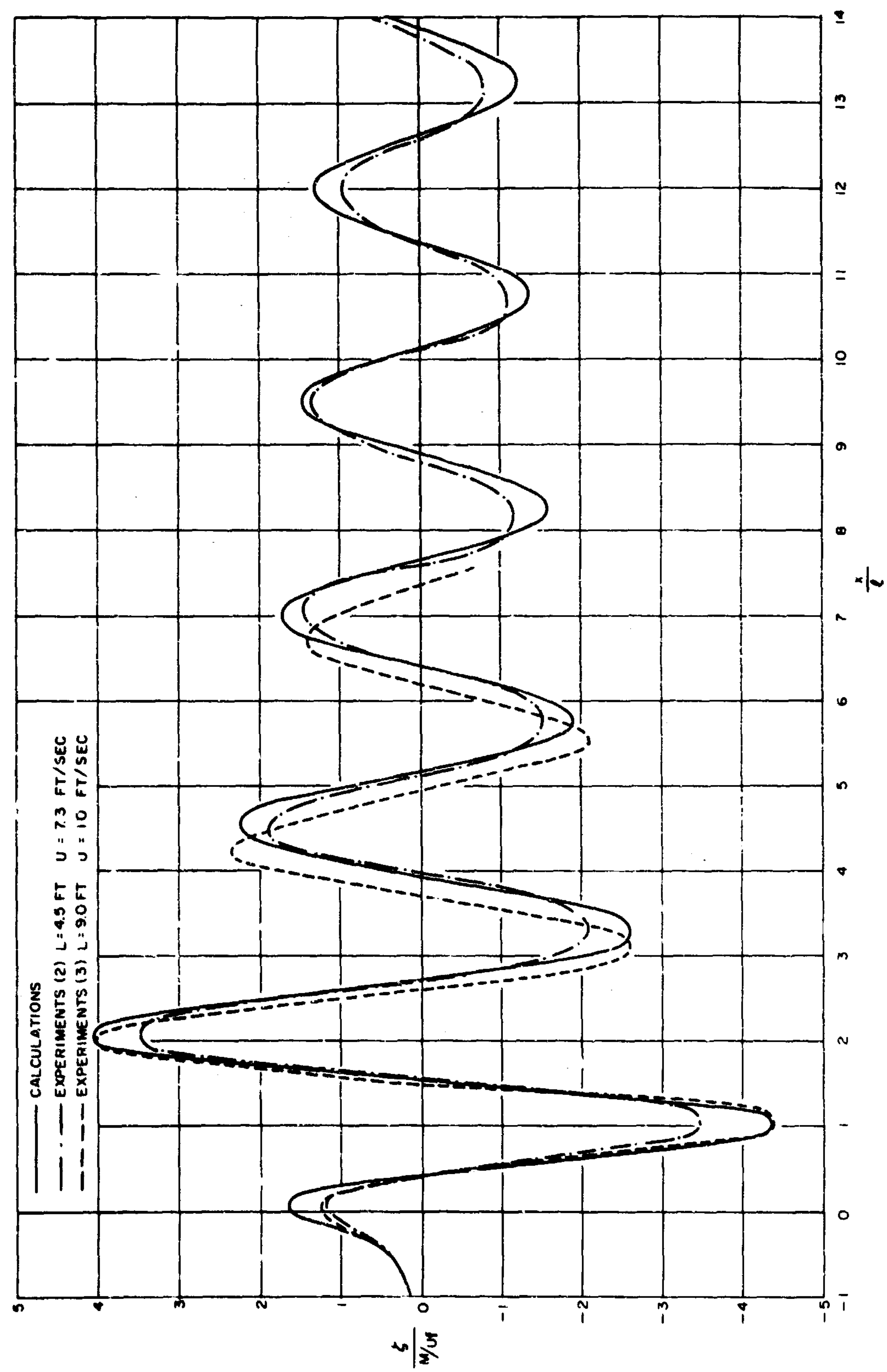

FIGURE 5- COMPARISON OF NON DIMENSIONAL THEORETICAL CALCULATIONS AND EXPERIMENTAL CENTERLINE WAVE PROFILE DUE TO 4.5 FT AND 9.0 FT LONG MODELS; $\ell/L = 1/3$

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